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Bond Graphs Approach to Modeling Thermal Processes

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Abstract— The paper presents the bond graphs approach to modeling thermal processes. The fundamental theory of bond graph modeling focused on thermal process is explored by considering the example of bath heated by hot fluid. Furthermore, a method to systematically build a bond graph model starting from an ideal physical model is given. In addition, procedure to generate mathematical equations and block diagram out of a causal bond graph is presented. Unlike the traditional mathematical modeling method, where the first step is to develop the equations for individual components, and then based on them the simulation scheme is created, the described method uses a reverse procedure.

Index Terms— Bond Graphs, Modeling dynamic systems, Thermal processes.

1 Introduction

BOND graphs represent a systematic method for modeling of systems in physics and engineering. Its advantage is that the method is based on a central physical concept - energy. This means that systems from different domains (electrical, mechanical, hydraulic, thermo-dynamic) are described in the same way. Bond graphs are a domain-independent graphical description of dynamic behavior of physical systems. It can automatically be translated into state-space equations. Furthermore, the bond graph modeling methodology also can be used for structural and causal analyses which are essential to design control and monitoring systems. As such, bond graph modeling may be considered as an integrated computer aided design tool in the field of system engineering.

The concept of bond graphs was first introduced by Henry M. Paynter, with the introduction of the junctions in April 1959. The method was further developed by Karnopp and Rosenberg in their textbooks (1968, 1975, 1983, 1990), such that it could be used in practice (Thoma, 1975; Van Dixhoorn, 1982). Thoma and Van Dixhoorn were the first to introduce bond graphs in Europe. By means of the formulation by Breedveld (1984, 1985) of a framework based on thermodynamics, bond-graph model description evolved to a systems theory. Over the last four decades there have been a lot of publications in the field of theory and application of Bond Graphs in different engineering domains**.**

The language of bond graphs aspires to express general class physical systems through power interactions. This method provides a uniform manner to describe the dynamical behavior for all types of physical systems. It illustrates the exchange power in a system, which is normally the product between the effort and flow. The power i.e. the product effort-flow has different interpretations in different physical domains. Yet, power can always be used as a generalized coordinate to model coupled systems residing in several energy domains. Besides this representation there is another one, in which the product effort-flow does not have the physical dimension of power, called pseudo bond graph (Thoma et al., 2000). Pseudo bond graphs are more suitable for chemical systems due to the physical meaning of the effort and flow variables.

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2 Bond Graph Methodology

Bond graphs are labeled and directed graphs, in which the vertices represent sub models and the edges represent an ideal energy connection between power ports. The edges are generally referred as bonds. These bonds are used to denote point to point connections between sub model ports. The key feature of bond graph modeling is the representation (by a bond) of exchanged power as the product of generalized effort (e) and generalized flow (f) with elements acting between these variables and junction structures (algebraic constraints) to reproduce the global model as interconnected subsystems. The power variables are the generalized flow and the generalized effort variables. For different domain the effort and flow variables used are different as per the type of domain. The generalized effort (e) is a potential variable (e.g. force, voltage, pressure, temperature, etc.); and the generalized flow (f) is a current variable (e.g. velocity, current, volume flow, entropy flow, etc.). Each process is described by a pair of variables, effort e and flow f, and their product is the power exchanged at the port.

On each bond, one of the variables must be the cause and the other the effect. This can be deduced by the relationship indicated by the arrow direction. Effort and flow causalities always act in opposite directions in a bond. The causality (cause and effect relationship) is the most important concept in bond graph theory. Indeed, the determination of causes and effects in the system is directly deduced from the graphical representation. In a bond graph model it is denoted by a cross-stroke at one end of a bond. The cause and effect relationship has physical meaning in most cases, but in general it represents which unknown variables can be calculated from which known variables. This leads to the description of bond graphs in the form of state–space equation.

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The concise bond graph notation (Fig.1) gives four pieces of information: the existence of physical link between two systems by the bond, the type of power (electric, mechanical, ...) by the power variables, the power direction by the half arrow and the causality by the stroke.

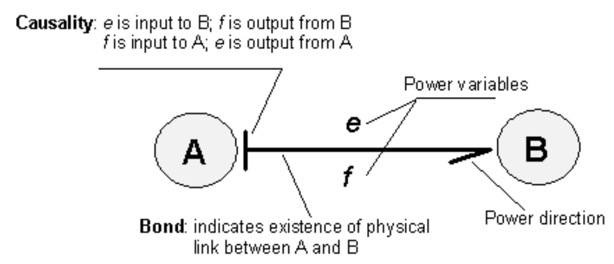

Fig. 1. Information given by a bond graph representation.

Besides the power variables, two additional physical quantities are very important for dynamic systems representation using bond graphs. These variables, called energy variables, are the generalized momentum *p* as time integral of effort and the generalized displacement *q* as time integral of flow. The energy variables are associated with state variables.

2.1 Bond graph elements

One of the advantages of bond graph method is that models of various systems belonging to different engineering domains can be expressed using a set of only nine elements. In bond graph language, two active elements (sources of effort Se and flow Sf), three generalized passive elements (inertia I, capacitor C and resistor R), two junctions (0 and 1) and two transducers (transformer TF and gyrator GY) are used to model any energetic process. Se and Sf elements are active elements because they supply power to the system. I, C, and R elements are passive elements because they convert the supplied energy into stored or dissipated energy. TF, GY, 0 and 1-junctions are junction elements that serve to connect I, C, R, and source elements and constitute the junction structure of bond graph model.

A classification of bond graph elements can be made up by the number of ports. Namely, the bond graph modeling describes interaction of subsystems. The interconnected part is referred as port. The standard form of a bond graph consists of 5 types of one port elements, 2 types of two port elements and 2 types of junctions as multiport elements to define their connections.

One port element is addressed through a single power port, and at the port a single pair of effort and flow variables exists. Ports are classified as passive ports and active ports. One port elements are represented by effort source (Se), flow source (Sf), capacitor (C), resistor (R) and inertia (I).

There are only two kinds of two port elements: transformer elements (TF) and gyrator elements (GY). As the name suggests, two bonds are attached to these elements. Transformers (TF) and gyrators (GY) are used to conserve power. The transformer relates flow-to-flow and effort-to-effort between its two bonds and the gyrator established relationship between flow-to-effort and effort-to-flow, again keeping the power on the ports same.

Multi ports elements include 0-junction and 1-junction. Junctions, sometimes called three ports, allow connecting the elements. The one port element and two ports element can be connected to the junction as well as the junction can be connected to the other junction. The 0-junction, also called effort junction, represents a node at which all efforts of the connecting bonds are equal. The 1-junction is also called flow junction and it represents a node at which all flows of the connecting bonds are equal.

2.2 Systematic procedure to build a bond graph model

The bond graph method makes possible the simulation of multiple physical domains, such as mechanical, electrical, thermal, hydraulic, etc., including combinations of these with each other, which can be treated as a unified set of elements. Flows $f(t)$ and efforts $e(t)$ should be identified with a particular variable for each specific physical domain which is working.

Once the system is represented in the form of Bond-graph, the state equations that govern its behavior can be obtained directly as the first order differential equations in terms of generalized variables, using simple and standardized procedures, regardless of the physical domain to which it belongs, even when interrelated across domains.

In general, there are four levels in the process of modeling of a dynamic system:

- Technological level, which represents the architecture of the system by the assembling different sub-models corresponding to different plant parts (pipe, heat exchanger, boiler,…).
- Physical level, at which the basic concepts of physics such as dissipation of energy, transformation, accumulation and so on are using, i.e. the modeling uses an energetic description of the physical phenomena.
- Mathematical level, which represents the mathematical model given by the mathematical equations that describe the system behavior.
- Algorithmic level, which indicates how mathematical models are calculated and directly is connected with information processing.

These four levels of modeling in bond graph representation correspond to:

- Word bond graph represents the technological level. It is used to make initial decisions about the representation of dynamic systems and indicates the major subsystems to be considered. The interconnection in word bond graph is realized by the power variables.
- A bond graph as a graphical model represents the physical level. The physical phenomena are represented by bond graph elements (storage, dissipation, inertia etc…).
- From this graphical model, but using deep physical knowledge, dynamic equations (algebraic or differential) are derived, which represents the mathematical level.
- Simulation program (how the dynamic model will be calculated) is shown by causality assignment, which

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represents the algorithmic level.

A step-by-step procedure to build the bond graph model of a system is shown in Fig.2.

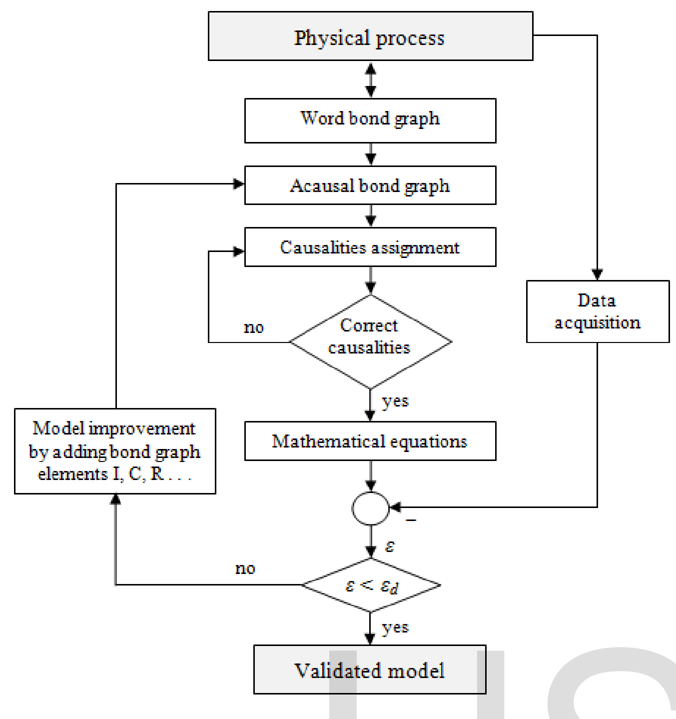

Fig. 2. Steps in bond graph modeling and validation.

3 Thermal Process Modeling

The fundamental theory of bond graph focused on thermal process is explored by considering the example of bath heated by hot fluid. The studied thermal system is given in Fig.3.

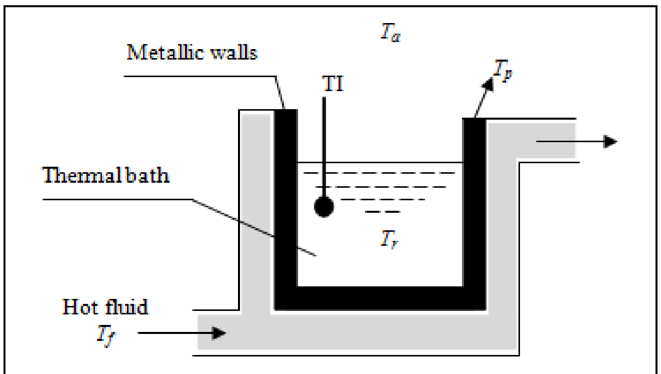

Fig. 3. Thermal system.

3.1 Process description

The process is heating of a thermal bath by hot fluid. The hot fluid is kept at a given temperature T_f. There is a heat exchanging between the hot fluid and the metallic walls which temperature is T_p. Heat is also leaving through the free surface to the environment. The environment supposed to be at a temperature T_a. Hence, there is a thermal resistor between the bath and the external environment. This thermal resistor may be linear (conduction only) or non-linear (conduction and radiation). The temperature sensor TI measures the bath temperature T_r.

3.2 Word bond graph

The used coupled power variables for the considered process are temperature T and thermal flow $\dot{Q}$. Namely, in the case of thermal conduction the pair (T, $\dot{Q}$) is used as pseudo bond graph power variables. The word bond graph is given in Fig.4.

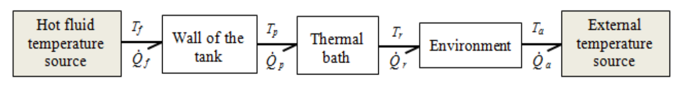

Fig. 4. Word bond graph of the thermal process.

3.3 Bond graph model

Bond graph as a graphical model represents the physical level of modeling. At first, modeling hypothesis for the studied process has to be assumed. In this case, only thermal capacity of the bath, C_r, is considered, while the thermal capacity of the wall, C_m, is taken as negligible. On the basis of assumed modeling hypothesis, the graphical representation of the physical phenomena can be made.

The bond graph can be realized using a following procedure:

- Fix reference axis for thermal flows (generally from high to low temperature).
- Place energy sources (temperature source Se:T_f and thermal flow source Sf: $\dot{Q}$.
- A 0-junction is connected with each different temperature, which corresponds to nodes of different potentials. If there is an energy storage in this junction, the C-element (C:C_r, here) has to be attached to the junction.
- Place a 1-junction between two 0- junctions or between 0-junction and thermal effort sources. Here, dissipation and heat transfer are modeled by attaching R-elements.
- Place sensors (De:T_r) and (Df: $\dot{Q}_p$).
- Connect all junctions taking into account the power directions.

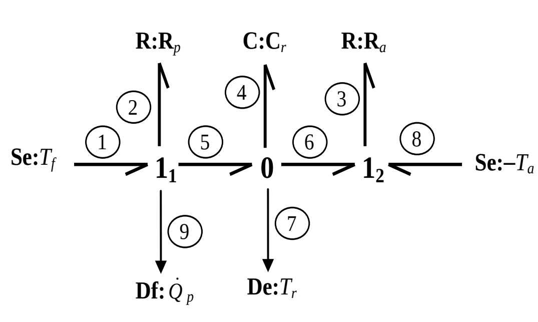

Fig. 5. Acausal bond graph of the thermal process neglecting the thermal capacity of the wall.

Hence, the bond graph model is constructed from an energetic consideration of the process. The temperature T_f is taken as an external effort source (Se:T_f). As it is shown in the word bond graph as well, there is a thermal transfer by conduction from the hot fluid T_f through the wall to the bath. This thermal transfer is modeled by R_p. Part of the transferred heat is accumulated in the thermal capacity of the water C_r and the other part is dissipated through R_a to the environment at the temperature T_a. The environment temperature T_a is modeled as a negative source (indicated by the orientation of the bond). All these effects are coupled by parallel and series junctions (0- and 1-junctions). The 0-junction expresses the common temperature points. The 1-junction represents the common thermal flows. The bond graph model is given in Fig.5, where all bonds are numbered in order to make it easy to read.

3.4 Causalities assignment

At this level of modeling, the computational structure of the model which is associated with block diagram structure is derived. The assignment of causalities is often guided by the necessity to resolve numerical problems. Hence, integral causality for dynamic elements is preferred for simulation, and derivative causality is purposed for model based fault detection and isolation. In the studied case, the model is required for simulation, so integral causalities for the bond graph model given in Figure 5a are imposed. It is start from the effort sources (Se:T_f and Se:T_a) because they are the compulsory causalities. After that, integral causality to C:C_r is assigned. In the next step, causality propagation through junction rules (e.g. only one causal stroke near a 0-junction) automatically forces causalities in the remaining elements R:R_p and R:R_a . In that way, the causal bond graph model is obtained and it is shown in Fig.6.

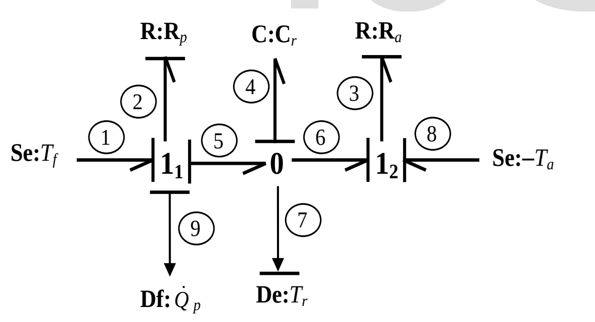

Fig. 6. Causal bond graph of the thermal process neglecting the thermal capacity of the wall.

3.5 Mathematical equations

Mathematical equations of the model are derived systematically from the causal bond graph. In accordance with assigned causality, there are:

- Structural equations which are the constitutive equations of 0-junctions and 1-junctions:

Junction $\mathbf{1_1}$: $$\begin{cases} e_2 = T_f - e_5 \\ f_1 = f_5 = f_9 = \dot{Q}_p = f_2 \end{cases}$$

Junction $\mathbf{0}$: $$\begin{cases} \dot{Q}_4 = f_4 = f_5 - f_6 \\ e_5 = e_7 = e_6 = T_r = e_4 \end{cases} \qquad (1)$$

Junction $\mathbf{1_2}$: $$\begin{cases} e_3 = e_6 - T_a \\ f_8 = f_6 = f_3 \end{cases}$$

- Constitutive behavioral equations of bond graphs elements:

R:R$_p$ element : $$f_2 = \frac{1}{R_p} e_2 = \frac{T_f - e_5}{R_p}$$

C:C$_r$ element: $$e_4 = \frac{1}{C_r}\int_0^t f_4 dt + e_4(0) = \frac{Q_4}{C_r} \qquad (2)$$

R:R$_a$ element : $$f_3 = \frac{1}{R_a} e_3 = \frac{1}{R_a}(e_6 - T_a)$$

In the above equations, C_r is the global thermal capacity. It is a parameter of C-element in the bond graph model. C_r depends on the specific heat capacity of the fluid in the bath at constant volume (c_v) and the total fluid mass (m_r), i.e. $C_r = m_r c_v$. Parameters R_p and R_a are thermal resistances of the hot fluid-bath interface, and the reservoir-environment interface, respectively. The thermal resistance R is reciprocal of the heat transfer coefficient K_c or conductance: $R = 1/K_c$.

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In the modeling of dynamic systems, state is a useful concept. The dynamics of a system, accompany the change of its state as time progress in those systems. The state-space equation and the output in the form:

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}x(t) + \mathbf{D}u(t)$$

can be derived from the bond graph. The state variable x, the control input u and the measurement y are respectively the energy variables in dynamic bond graph element (C), the source (Se) and the effort detector (De). There is only one C-element in integral cusalyty since the hydraulic energy is neglected and the the mass of the fluid is assumed to ne constant. Therefore, there is only one state variable – acomulated thermal energy in the tank. The inputs are thermal effort sourcesand the measured variables are given by the detectors.

$$x = Q_4 \equiv Q_r \ , \quad u = \begin{bmatrix} T_f & T_a \end{bmatrix}^T \ , \quad y = \begin{bmatrix} \dot{Q}_p & T_r \end{bmatrix}^T \qquad (3)$$

Therefore, the obtained state-space model is:

$$\dot{Q}_r(t) = -\left(\frac{1}{R_p C_r} + \frac{1}{R_a C_r}\right) Q_r(t) + \left(\frac{1}{R_p} + \frac{1}{R_a}\right) \begin{bmatrix} T_f(t) \\ T_a(t) \end{bmatrix}$$

$$y(t)=\begin{bmatrix}\dot{Q}_p(t)\\ T_r(t)\end{bmatrix}=\begin{bmatrix}-\dfrac{1}{R_pC_r}\\ \dfrac{1}{C_r}\end{bmatrix}Q_r(t)+\begin{bmatrix}\dfrac{1}{R_p} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}T_f(t)\\ T_a(t)\end{bmatrix} \qquad (4)$$

3.6 Simulation

Block diagram for simulation can be easily developed on the basis of (1) and (2). It is given in Fig.7.

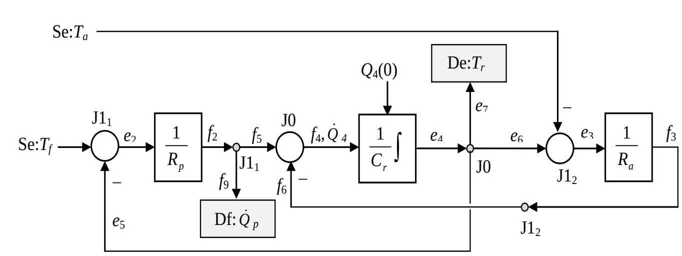

Fig. 7. Block diagram for simulation.

3.7 Model modification

In case there is a need to take into consideration some physical phenomenon which has been neglected in the first version, the bond graphs element has to be added in the bond graph model. There is no need to write the equations from the begining.

The thermal capacity C_m of the bath wall was neglected at the first version. In case to take it into consideration, since one part of the heat transfer from the hot fluid to the bath wall is accumulated by the thermal capacity of the wall, the thermal capacity of the bath wall modeled by C:C_m has to be added as a new element in the previous bond graph model. The modified bond graph model is given in Fig.8.

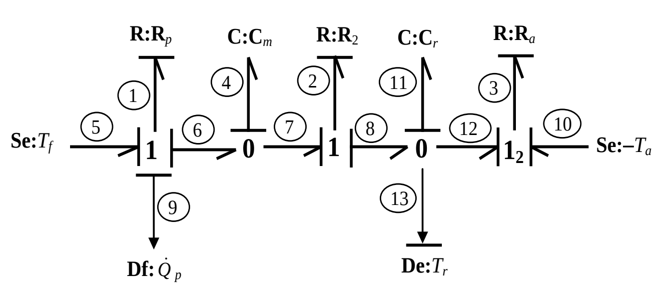

Fig. 8. Revised bond graph model.

In this case, the system has two C-elements and three R-elements. The energy storage in the bath wall Q_m, as well as the enrgy storage from the heat bath itself Q_r have been considered, so the system has two state variables. The state-space model, obtained from the constitutive equations of two C-elements and three R-elements, is given with the following equatios:

$$\dot{x}(t)=\begin{bmatrix}\dot{Q}_m\\ \dot{Q}_r\end{bmatrix}=\begin{bmatrix}-\left(\dfrac{1}{R_1C_m}+\dfrac{1}{R_2C_m}\right) & \dfrac{1}{R_2C_r}\\ \dfrac{1}{R_2C_m} & -\left(\dfrac{1}{R_2C_r}+\dfrac{1}{R_aC_r}\right)\end{bmatrix}\begin{bmatrix}Q_m\\ Q_r\end{bmatrix}+$$
$$+\begin{bmatrix}\dfrac{1}{R_1} & 0\\ 0 & -\dfrac{1}{R_a}\end{bmatrix}\begin{bmatrix}T_f\\ T_a\end{bmatrix}$$

(5)

$$y(t)=\begin{bmatrix}\dot{Q}_p\\ T_r\end{bmatrix}=\begin{bmatrix}-\dfrac{1}{R_1C_m}\\ \dfrac{1}{C_r}\end{bmatrix}\begin{bmatrix}Q_m\\ Q_r\end{bmatrix}+\begin{bmatrix}\dfrac{1}{R_1} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}T_f\\ T_a\end{bmatrix}$$

4 Conclusion

Bond graphs represent a systematic method for the modeling of systems in physics and engineering. Its advantage is that the method is based on a central physical concept - energy. Bond graphs are a domain-independent graphical description of dynamic behavior of physical systems. It can automatically be translated into state-space equations. The bond graph modeling methodology also can be used for structural and causal analyses which are essential to design control and monitoring systems. As such, bond graph modeling may be considered as an integrated computer aided design tool in the field of system engineering.

The paper explored the fundamental theory of bond graph focused on thermal process by considering the example of bath heated by hot fluid. Furthermore, a method to systematically build a bond graph model starting from an ideal physical model is presented. In addition, procedure to generate mathematical equations and block diagram out of a causal bond graph is also considered.

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